A Model of Monetary Singleness *

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20 July 2025

Abstract

Rapid innovation in digital payments and the advent of new forms of privately issued digital money have increased interest in the concept of singleness of money. This paper provides an analytical framework for studying the singleness of money consisting of a three-period banking model where banks choose both the unit of account of their debt and whether it can be used as a medium of exchange. The paper suggests that small deviations from a common unit of account may still be consistent with the efficient allocation but that inefficient equilibria are more likely to occur if the newly introduced forms of digital money are issued by private entities with distinct business models from incumbent financial institutions. The model suggests that cash also has an important role to play in promoting the singleness of money by providing a means of payment in the absence of interbank transfers.

Keywords: Banking, Money, Singleness, Unit of account

JEL codes: E41, E42, E58

*The views expressed in this paper are those of the author and do not necessarily reflect the position of the Bank of England or its committees.

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1 Introduction

The accelerating pace of innovation in digital payments and the proliferation of privately issued digital monies have rekindled interest among policy makers in the concept of singleness of money. At its core, singleness refers to the idea that all forms of money in an economy should trade at par and share a common unit of account. This property underpins the seamless functioning of monetary exchange, yet its preservation is increasingly challenged by the emergence of new monetary instruments with diverse technological and institutional foundations.

Despite its practical importance, the academic literature has only recently begun to grapple with the implications of singleness in a world of programmable money, stablecoins, and tokenised deposits. Key questions remain unresolved. Why is singleness desirable in the first place? Should policymakers tolerate small deviations from par exchange, or is "approximate singleness" a contradiction in terms? At stake is whether monetary authorities should enforce strict par convertibility across monetary instruments to preserve the integrity of the unit of account, or whether limited deviations from par can be tolerated in pursuit of financial innovation. Garratt and Shin (2023) argue that even minor deviations from par can introduce frictions that cascade through the economy, undermining the coordinating role of money. Others, such as Chiu and Monnet (2025), take a more permissive view, suggesting that modest departures from par may be a tolerable price for the potential innovation that digital money may bring.

A second set of questions concerns the mechanisms through which singleness can be maintained. The role of regulation and central bank reserves in supporting the financial system is relatively well supported in the literature. For example, Gorton (2020) provides a summary of how regulation prevents bank runs. The role of retail fiat money, that is, cash, in preserving the singleness of money seems less well established. A growing number of papers such as Armelius et al. (2020), BIS (2003), and Rivadeneyra et al. (2024) suggest that cash has a role to play in preserving singleness by enhancing trust in the financial system and through other behavioural mechanisms.

This paper develops a tractable analytical framework to study these questions. Building on the unit of account model of Doepke and Schneider (2017), I construct a three-period banking model in which banks choose both the unit of account in which their liabilities are denominated and whether those liabilities can be used as a medium of exchange. The model captures the trade-offs banks face when deciding the degree of interoperability between different forms of money. Within this framework, if the unit of account of a bank's debt differs from the unit of account of its asset base, then exchange rate risk constrains the bank's ability to borrow. An individual bank can eliminate this exchange rate risk by choosing to issue debt in the same units as its production. However, if two banks choose different units of account, allowing interbank transfers reintroduces this exchange rate risk. Banks must trade off the benefits of interoperability against the exchange rate risk this generates.

The model has several important policy implications. First, it shows that, while small deviations from a common unit of account may be consistent with efficiency, the likelihood of inefficient equilibria increases when monetary instruments are backed by heterogeneous asset portfolios. This is particularly relevant in the context of stablecoins and other privately issued digital monies, which may be backed by assets that differ significantly from those held by traditional banks. Second, the model highlights the stabilising role of cash as a universally accepted medium of exchange that can serve as a backstop when full interoperability of private monies is impaired. Third, it suggests that, while a common unit of account often coincides with the first-best equilibrium, it is not a necessary condition for efficiency. This opens the door to a more nuanced policy stance that tolerates limited monetary diversity without sacrificing welfare.

This paper is related to several strands of literature. First, there is the growing literature on the role of money as a unit of account. Doepke and Schneider (2017) develop a model in which agents enter into non-contingent contracts across the credit chain. Their framework shows that the use of a common unit of account reduces exposure to relative price risk and improves welfare. Drenik et al. (2022) examine currency choice in contracts and show that agents' preferences over units of account depend on both the co-movement of currency values and their consumption needs. Zúñiga (2023) embeds the unit of account in a dynamic monetary model and suggests that inflation targeting supports transactional efficiency, while price-level targeting better preserves the unit of account function of money. Distinct from the existing literature, this paper models the incentives of private money issuers to deviate from a common unit of account and allows for differing degrees of interoperability between private monies.

A second strand of literature focusses on the inherent instability of privately issued money. Sanches (2016) shows that competitive private money systems are prone to self-fulfilling collapses and argues that fiscal interventions may be necessary to ensure equilibrium determinacy and monetary stability. Fernández-Villaverde and Sanches (2019) model currency competition among privately issued fiat currencies and show that such systems typically fail to deliver efficient allocations. Even when price stability is achieved, the absence of a coordinating mechanism leads to fragmentation and inefficiency. Calomiris and Kahn (1991) provide a theoretical rationale for demandable debt, arguing that it disciplines banks through the threat of withdrawal, while Donaldson and Piacentino (2022) study the failure of demandable debt to circulate.

A third strand of literature examines how digital innovation, particularly tokenisation and platform-based money, affects the singleness of money. Brunnermeier et al. (2021) suggest that digital innovation could result in a situation in which a national currency is replaced by the currency of a digital platform, a process they refer to as "Digital Dollarization". They suggest that allowing digital forms of money to be convertible to a central bank digital currency (CBDC) would uphold the unit of account status of public money. Garratt and Shin (2023) distinguish between digital bearer instruments, such as stablecoins and tokenised deposits. They argue that bearer instruments are prone to deviations from par due to settlement frictions and issuer-specific credit risk, whereas tokenised deposits settled in central bank money preserve singleness. Chiu and Monnet (2025) explore the implications of programmable digital currencies for monetary uniformity. They show that programmability can compromise singleness by introducing heterogeneity in liquidity and transferability across money. Their model highlights a trade-off between commitment benefits and informational frictions, suggesting that singleness may not always be socially optimal. Ozdenoren et al. (2025) examine platform-issued money and show that platforms may issue their own currencies to extract seigniorage and reduce transaction costs. However, this can lead to fragmentation and reduced market tightness.

The remainder of this paper is organised as follows. Section 2 presents the model. Section 3 describes the possible equilibria. Section 4 provides some numerical examples of equilibria. Section 5 discusses the effectiveness of various tools available to policy makers to promote efficient outcomes, and Section 6 concludes.

2 The Model

In this section, I present the model.

2.1 Environment

There are two goods $i \in \{A, B\}$ that are both the output of production and consumption goods. Time is discrete and there are three dates, 0, 1, and 2.

There are two regions. Within each region, there are three agents: a bank, a labourer, and an artisan. At date 0, the bank and labourer located in the same region enter into a bilateral contracting arrangement where the labourer provides labour input n to the bank in exchange for a debt contract, to be specified below. At date 1, they have a probability to meet an artisan. Should a labourer and an artisan meet, an artisan is able to expend x units of labour effort to make x units of a customised good which can be consumed at date 2 only by the labourer for whom it was made. It cannot be traded on the open market. In date 2, the output of the bank's investment is realised and a spot market opens in which the two goods A and B can be traded. As in Doepke and Schneider (2017), the price vector $\mathbf{p}$ is exogenous and random with convex and compact support $\mathbf{P} \subset \mathbb{R}^2_{>0}$.

All agents receive instantaneous utility $v(\mathbf{c})$ from receiving a vector $\mathbf{c}$ of goods A and B at date 2 where $v(\cdot)$ is homogeneous of degree 1. The bank's only source of utility at date 2 is from consumption of the tradable goods.

With probability $\alpha \geq 0$ the labourer meets an artisan in their own region, with probability $\beta \geq 0$ the labourer meets an artisan located in the other region, and with probability $1-\alpha-\beta \geq 0$ the labourer does not meet an artisan. It is assumed that these probabilities are symmetric for all regions. The utility of an artisan that works x units of time to produce a customised good for a labourer and consumes a vector $\mathbf{c}$ of tradable goods is

$$v(\mathbf{c}) - x.$$

The utility of a labourer that provides n units of labour to the bank at date 0, receives x units of a customised good from an artisan and consumes a vector $\mathbf{c}$ of tradable goods is

$$v(\mathbf{c}) + (1+\gamma)\, x - n,$$

where $\gamma > 0$ implies that there are gains from trade between artisan and labourer. While banks are able to issue debt contracts to labourers, labourers are unable to issue debt contracts of their own. Trade between labourers and artisans can only take place if the bank debt held by the labourers can be transferred to artisans using them as means of payment.

Each bank produces a positive vector of goods denoted by $\mathbf{y}_j$ according to the following production function

$$\mathbf{y}_j = n_j^v \hat{\mathbf{y}}_j, \tag{1}$$

where n_j is the scalar quantity of labour supplied to Bank j by the labourer located in the same region as Bank j. The parameter $v \in (0,1)$ is such that the bank's production function has diminishing returns to the labour input. Each Bank produces a specific ratio of goods A and B captured by the unit vector $\hat{\mathbf{y}}_j$, which I refer to as the unit of production of Bank j. Each element of $\hat{\mathbf{y}}_j$ is weakly positive, denoting that production of each good is non-negative.

As in Doepke and Schneider (2017), I normalise prices in the following way. First, prices are normalised so that the price vector $\mathbf{p}$ satisfies $\tilde{P}(\mathbf{p}) = 1$ for all $\mathbf{p} \in P$ where $\tilde{P}(\mathbf{p})$ is the expenditure function at a utility level of 1 and

$$\tilde{P}(\mathbf{p}) = \min_{c} \{\mathbf{p}'\mathbf{c}\}$$

subject to $v(\mathbf{c}) \geq 1$. This normalisation along with the assumption that v is homogeneous of degree 1 simplifies the analysis, since the indirect utility of consuming a bundle $\mathbf{y}$ of goods A and B at date 2 for a given price realisation is $\mathbf{p}'\mathbf{y}$. Second, the expected price of each good $i \in \{A, B\}$ is equal to 1: $E(p^i) = 1$.

2.2 Bank Debt

At date 0, the bank and labourer located in region j negotiate a bilateral contract where the labourer agrees to provide n_j units of labour to the bank at date 0 in exchange for bank debt $\mathbf{b_j}$ which is a promise to deliver a bundle of goods A and B at date 2. It will be useful to decompose bank debt into a magnitude $B_j > 0$ and a unit of account, $\hat{\mathbf{b}}_\mathbf{j}$ such that $\mathbf{b_j} = B_j \hat{\mathbf{b}}_\mathbf{j}$ and the unit of account $\hat{\mathbf{b}}_\mathbf{j}$ is the unit vector defined by

$$\hat{\mathbf{b}}_\mathbf{j} = \frac{\mathbf{b}_j}{\|\mathbf{b}_j\|_1}. \tag{2}$$

In addition to agreeing on the size of the debt, B_j and its unit of account $\hat{\mathbf{b}}_\mathbf{j}$, the bank and the labourer must also decide whether the debt can be transferred to another agent. Should the debt be transferable, it can be used as a means of payment and thus can facilitate trade between labourers and artisans. I allow for the possibility that transferability is limited only to artisans located in the same region as the bank and the artisan. I also allow for the bank to specify a fee should the labourer wish to transfer the debt to another agent. The fee $\chi_{j,k} \geq 0$ is assumed to be a fraction of the debt paid to bank j should the labourer wish to transfer the debt to an agent located in region k. The fee is allowed to vary depending on whether the transfer is made to an agent located in the same region as the bank or not.

I assume that the bank sets the terms of the contract by making a take-it-or-leave-it offer to the labourer, subject to feasibility constraints which are set out below. I also assume that banks are able to fully commit to the terms of the contract agreed at date 0.

At date 2, the bank must have sufficient resources to make the promised payment to the labourer for all possible price realisations $\mathbf{p}$. A debt that is not transferred to another agent is feasible if it satisfies the following condition

$$\mathbf{p}'\left(n^v\hat{\mathbf{y}}_\mathbf{j} - B_j\hat{\mathbf{b}}_\mathbf{j}\right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \tag{3}$$

At date 1, labourers may meet artisans and attempt to trade with them. If the bank debt issued by j is transferable, the labourer can at date 1 request bank j to transfer part of its debt s to an artisan. I assume that all debts issued to artisans in region j are denominated in the same unit of account as the debt issued to the labourer in region j, and therefore bank j can allow intrabank transfers at no additional cost. As $\gamma > 0$ there are gains from trade between artisans and labourers, and bank j will benefit from allowing intrabank transfers. Although bank j can set an interbank fee $\chi_{j,j}$, setting $\chi_{j,j} > 0$ will not affect the feasibility constraint set out in equation (3) as this must also be satisfied if the labourer does not meet an artisan.

If the labourer requests bank j to transfer a fraction of its debt $s \in [0, 1]$ to an artisan located in region $k \neq j$ then the bank in region k must agree to take on this debt from bank j, effectively requiring a functioning interbank market. This interbank market is structured as follows. First, it is assumed that the bank that initiates the trade makes a take-it-or-leave-it offer to the receiving bank consisting of an interbank debt issued to the receiving bank $\mathbf{t}_{j,k}$ and the amount of debt to be transferred $sB_j\hat{\mathbf{b}}_j$. The transfer fee consists of a bundle of tradable goods and can be decomposed into the transfer scale, $T_{j,k}$, and the unit of account for the transfer, $\hat{\mathbf{t}}_{j,k}$. The unit of account of the transfer $\hat{\mathbf{t}}_{j,k}$ is a unit vector and the decomposition is such that $\mathbf{t}_{j,k} = T_{j,k}\hat{\mathbf{t}}_{j,k}$.

If the receiving bank, k, accepts the terms of the transfer, it receives interbank debt $T_j\hat{\mathbf{t}}_{\mathbf{j},\mathbf{k}}$ from the transferring bank, j. Bank k then takes on a fraction s of bank j's debt net of the cost of the transfer cost $\chi_{j,k}$. As it is assumed that bank k uses the same unit of account for all debt issued to labourers and artisans in region k, bank j' debt is exchanged for bank k debt at the expected exchange rate at date 1 which is $\frac{E\left[\mathbf{p}'\hat{\mathbf{b}}_\mathbf{j}\right]}{E\left[\mathbf{p}'\hat{\mathbf{b}}_\mathbf{k}\right]}$. Given the assumption that $E[p_i] = 1$ for all $i \in \{A, B\}$ and $\hat{\mathbf{b}}_\mathbf{j}$ and $\hat{\mathbf{b}}_\mathbf{k}$ are unit vectors, it follows that $\frac{E\left[\mathbf{p}'\hat{\mathbf{b}}_\mathbf{j}\right]}{E\left[\mathbf{p}'\hat{\mathbf{b}}_\mathbf{k}\right]} = 1$.

It must also be feasible for bank k to accept the debt transfer and to pay its obligation to the artisan in region k given all possible price realisations $\mathbf{p}$. To simplify the analysis, I assume that the interbank debt received by bank k must be sufficient to cover its liabilities to the artisan in region k. Thus, it is feasible for the bank k to accept a bank transfer if the following equation holds

$$\mathbf{p}'\left(T_j\hat{\mathbf{t}}_{\mathbf{j},\mathbf{k}} - s\left(1 - \chi_{j,k}\right)B_j\hat{\mathbf{b}}_\mathbf{k}\right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \tag{4}$$

A transfer of debt between bank j and bank k also changes the financial obligations of bank j. A fraction s of bank j's debt to the labourer in region j is cancelled, but a new interbank debt obligation to bank k is created. For the interbank transfer to be feasible, bank j must be able to repay its obligations to both the labourer in region j and the bank in region j from its output given all possible price realisations. Thus, the following must hold

$$\mathbf{p}'\left(n^v\hat{\mathbf{y}}_{\mathbf{j}} - (1-s)\,B_j\hat{\mathbf{b}}_{\mathbf{j},\mathbf{k}} - T_j\hat{\mathbf{t}}_{\mathbf{j},\mathbf{k}}\right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \tag{5}$$

As the bank initiating the transfer, in this case bank j, has all the bargaining power, it follows immediately from equation (4) that it is optimal for bank j to ensure bank k makes zero profit from the transfer in every possible price realisation. This occurs if the following holds

$$T_j\mathbf{p}'\hat{\mathbf{t}}_{\mathbf{j},\mathbf{k}} = s\,(1-\chi_{j,k})\,B_j\mathbf{p}'\hat{\mathbf{b}}_{\mathbf{k}} \quad \forall \mathbf{p} \in \mathbf{P}. \tag{6}$$

For this to hold in every possible price realisation, it must be the case that the interbank loan has the same unit of account as the debt bank k issues to the region k artisan. That is, it must be the case that $\hat{\mathbf{t}}_{\mathbf{j},\mathbf{k}} = \hat{\mathbf{b}}_{\mathbf{k}}$. The size of the interbank transfer is thus set to ensure that equation (4) binds for all price realisations.

Should the units of account differ between banks j and k, $\hat{\mathbf{b}}_{\mathbf{j}} \neq \hat{\mathbf{b}}_{\mathbf{k}}$, allowing for debt to be transferred between the two banks creates additional uncertainty relating to the realisation of the value of the liabilities created in the transfer. If the receiving bank k takes on some of this risk, then the feasibility constraint, equation (4), must be slack in some situations resulting in bank k earning strictly positive profit in expectation. From the perspective of bank j it is optimal for bank j to fully insure bank k from the uncertainty that results from the mismatch of the unit of account. In this case, equation (4) will always hold. Allowing debt to be fully transferable for any $s \in [0, 1]$ and substituting in the optimal interbank contract offered by bank j, equation (5) can be rewritten as

$$\mathbf{p}'\left(n^v\hat{\mathbf{y}}_j - (1-\chi_{j,k})\,\hat{\mathbf{b}}_{\mathbf{k}}B_j\right) \geq 0 \quad \forall \mathbf{p} \in \mathbf{P}. \tag{7}$$

Thus, if bank j agrees to allow their banks to transfer their debt, then a debt contract is feasible if it satisfies both equation (3) and equation (7). If the units of account differ across banks, then allowing debt to be transferable between banks results in additional constraints on the borrowing capacity of the banks. This can be compensated for to some extent by charging a fee on transfers of debt between banks $\chi_{j,k}$.

In the model, each bank chooses a unit of account vector $\hat{\mathbf{b}}_j$ in which its liabilities are denominated. If this vector diverges from the bank's output vector $\hat{\mathbf{y}}_j$, then the bank's ability to meet its obligations depends on the realisation of the relative price vector $\mathbf{p} \in \mathbf{P}$.

As it is assumed that these obligations must be met for all possible price realisations, it is useful to define a 'worst' price realisation from the perspective of bank j. There exists a price vector $\mathbf{p_j} \in \mathbf{P}$ that minimises the value of bank j's period 2 endowment $\mathbf{p'\hat{y}_j}$. This can be formally defined as the point where $\mathbf{p_j} = \arg\min_{\mathbf{p}\in\mathbf{P}} \mathbf{p'\hat{y}_j}$.

To further simplify the problem, define the variable $\epsilon_{j,k}$ as the relative cost of settling obligations denominated in the unit of account of bank k from bank j's perspective is

$$\epsilon_{j,k} = \frac{\mathbf{p}_j'\mathbf{\hat{b}_k}}{\mathbf{p}_j'\mathbf{\hat{y}_j}} \tag{8}$$

Intuitively, $\epsilon_{j,k}$ measures the number of units of bank j's output (valued at the worst-case price vector $\mathbf{p}_j$) required to purchase one unit of bank k's unit of account. Due to the definition of $\mathbf{p}_j$, the smallest possible value that $\epsilon_{j,k}$ can take is 1, which occurs if the unit of account of bank k's debt is identical to bank j's unit of account, $\mathbf{\hat{b}_k} = \mathbf{\hat{y}_j}$. When $\epsilon_{j,k} > 1$, bank j faces some risk that stems from the difference between the units of account of its output and its debts. As a consequence, the bank may find it costly to honour obligations denominated in $\mathbf{\hat{b}}_k$.

Equation (3) can then be written as

$$n_j^v \mathbf{\hat{y}_j} \frac{1}{\epsilon_{j,j}} - B_j \geq 0, \tag{9}$$

where $\epsilon_{j,j}$ captures the difference between bank j's unit of production and the unit of account on its debt.

Similarly, equation (7) can be written as

$$n_j^v \frac{1}{\epsilon_{j,k}} - (1 - \chi_{j,k}) B_j \geq 0. \tag{10}$$

2.3 Decentralised Trade

Allowing debt to be transferable between banks imposes an additional feasibility constraint on bank debt should banks have different units of accounts. However, allowing bank debt to be transferable also allows it to be used as a means of payment, thus allowing for gains from the decentralised trade between labourers and artisans. It follows that for banks and labourers to agree that debt is transferable, the benefits must outweigh the costs.

Consider the case of a labourer in region j that provides n_j units of labour to bank j in exchange for bank debt $\mathbf{b_j} = B_j\mathbf{\hat{b}_j}$. If the labourer does not meet and trade with an artisan at date 1, then at date 2 the bank provides the promised bundle of tradable goods. Given the price normalisation and the assumption that $v(\cdot)$ is homogeneous of degree 1, the utility of the labourer can be written as

$$E[\mathbf{p}'\mathbf{b}_j] - n_j = B_j - n_j. \tag{11}$$

This is also the expected utility of a labourer that is paid with non-transferable debt and thus is unable to trade with artisans it meets at date 1.

Consider the case where the labourer in region j meets an artisan in some region k and where if $k = j$ the artisan and the labourer are located in the same region. For now, Assuming the bank debt is transferable between region j and k, the labourer can use its bank debt as a means of payment, subject to the transfer fee agreed to previously $\chi_{j,k}$ which may vary depending on the region to which the debt is to be transferred. The labourer wishes to make a take-it-or-leave-it offer to the artisan that offers a fraction $s \in [0, 1]$ of their debt in exchange for x_k units of the customised good. The labourer thus chooses s and x_k to maximise

$$\max_{s,x_k} (1-s)\, E[\mathbf{p}'\mathbf{b}_j] + (1+\gamma)\, x_k - n_j, \tag{12}$$

subject to the artisan's participation constraint that can be written as

$$s\,(1-\chi_{j,k})\, E[\mathbf{p}'\mathbf{b}_j] - x_k \geq 0. \tag{13}$$

It is straightforward to see that the solution to the labourer's problem will be a corner solution where the labourer will either maximise the amount of trade by setting $s = 1$ and choosing x_k such that the artisan's participation constraint binds, or they will set $s = x_k = 0$ and not trade at all.

Given $\gamma > 0$ and there are gains from trade, trade will take place as long as the fee is not too large. Specifically, the fee must be such that

$$(1+\gamma)\,(1-\chi_{j,k}) \geq 1. \tag{14}$$

Assuming the transfer fee is sufficiently low, the artisans expected utility from tradable debt is

$$\begin{aligned} U_j =& \alpha\,(1+\gamma)\,(1-\chi_{j,j})\, E[\mathbf{p}'\mathbf{b_j}] \\ &+ \beta\,(1+\gamma)\,(1-\chi_{j,k})\, E[\mathbf{p}'\mathbf{b_j}] \\ &+ (1-\alpha-\beta)\, E[\mathbf{p}'\mathbf{b_j}] - n_j, \end{aligned} \tag{15}$$

where α is the probability that the labourer from region j meets an artisan also from region j while with probability β, the labourer meets a foreign artisan from region $k \neq j$.

Using the price normalisation the above can be rewritten as

$$U_j = \left[(1+\gamma)\left(\alpha\left(1-\chi_{j,j}\right)+\beta\left(1-\chi_{j,k}\right)\right)+(1-\alpha-\beta)\right]B_j - n_j, \tag{16}$$

where B_j is the size of the debt the labourer receives from bank j. The above equation will enter as a constraint in the Bank's optimisation problem which will be detailed later. However, it is worth noting that the quantity of labour that a labourer is willing to supply will be larger in cases where the debt is transferable and where the transfer fees are sufficiently low. Also, since labourers are risk neutral, the unit of account of bank j's debt does not directly enter the utility function of the labourer.

2.4 Timeline

First, at date 0 the bank in region j makes a take-it-or-leave-it offer of a promise to pay the labourer in region j a bundle of goods $\mathbf{b_j}$ in exchange for labour input n_j. The bank specifies under which conditions the debt can be transferred to another agent and specifies a transfer fee that is payable if the debt is transferred. If the labourer accepts, the bank receives labour input to its production function and the labourer becomes a debtholder.

Second, at date 1 the labourers may randomly meet an artisan. A labourer who meets an artisan and holds transferable bank debt makes a take-it-or-leave-it offer to the artisan, specifying the amount of bank debt to transfer in exchange for a quantity x of a custom good. If bank j receives a request to transfer its debt to a bank located in region k, bank j makes a take-it-or-leave-it offer to bank k specifying the size of the debt to be issued by bank k in exchange for an interbank debt issued by bank j.

Finally, at date 2, the bank's production is realised, and the banks make good on their promised payments. A Walrasian goods market opens and the prices of the two tradable goods $\mathbf{p}$ are realised.

2.5 First Best

A useful benchmark to consider is the efficient first best allocation. First, note that it will be optimal for labourers and artisans to trade as much as possible whenever they meet. Given the normalisation of the price vector $\mathbf{p}$, the marginal benefit of a unit of the bank's output is equal to 1 if the labourer does not meet an artisan and $1+\gamma$ if they do. The social planner problem can be written as

$$\mathcal{W} = \max_{\{n_j\}_j} \sum_j \left[(1+\gamma(\alpha+\beta)) E[\mathbf{p}'\hat{\mathbf{y}}_\mathbf{j}] n_j^v - n_j \right], \tag{17}$$

where due to the price normalisation $E[\mathbf{p}'\hat{\mathbf{y}}_\mathbf{j}] = 1$. The planner chooses the optimal scale of the bank's project such that the marginal benefit from an additional unit of labour equals the marginal cost of the labourer providing it. The optimal scale of bank j's project is thus given by

$$n_j^* = \left[v(1+\gamma(\alpha+\beta)) \right]^{\frac{1}{1-v}}, \tag{18}$$

while optimal welfare is

$$\mathcal{W} = \sum_j (1-v) \left[v(1+\gamma(\alpha+\beta)) \right]^{\frac{v}{1-v}}. \tag{19}$$

Aggregate welfare is increasing in the probability that artisans and labourers trade, α and β, as well as the gains from trade γ.

3 Equilibrium

In this section, I study the profit maximising contract offered by banks to labourers in the presence of contracting frictions. Bank j chooses first, under which conditions the debt can be transferred, the quantity of labour n_j provided by the labourer, the size of the debt B_j, and the unit of account the debt is denominated in $\hat{\mathbf{b}}_\mathbf{j}$.

As discussed in the previous section, there is no cost to bank j in allowing its debt to be transferred to an artisan located in region j, and I focus on cases where intrabank transfers are allowed. The choices bank j makes regarding the conditions of transfer are first, the fee on intrabank transfers $\chi_{j,j}$, second whether to allow interbank transfers and finally if interbank transfers are allowed, what fee $\chi_{j,k}$ to charge.

3.1 Autarky

First, consider the contract terms that would be offered by bank j if they choose not to allow interbank transfers. This corresponds to an autarkic equilibrium in which the liabilities of each bank circulate only within its own region. Bank j then chooses labour n_j, the size of the debt B_j, the unit of account of the debt, which can be summarised by the relative price $\epsilon_{j,j}$, and the intrabank transfer fee $\chi_{j,j}$. Bank j chooses these to maximise their expected profit, conditional on not allowing interbank transfers.

The bank's optimisation problem is to maximise it profit

$$V_j = \max_{\{n_j, B_j, \hat{\mathbf{b}}_{\mathbf{j}}, \chi_{j,j}\}} n_j^v - (1 - \alpha\chi_{j,j}) B_j, \tag{20}$$

subject to the participation constraint of the labourer, the promise-keeping constraint on its debt, and a maximum intrabank transfer fee that would be paid by the labourer.

The participation constraint of the labourer consists in ensuring the labourer's expected utility is non-negative, $U_j \geq 0$. From equation (16) and noting that setting bank debt to be non-transferable between regions is equivalent to setting $\beta = 0$ yields the following participation constraint

$$[\alpha (1 + \gamma) (1 - \chi_{j,j}) + 1 - \alpha] B_j \geq n_j. \tag{21}$$

The bank must ensure that the promise-keeping constraint defined by equation (9) ensures that it is able to meet its debt obligations to the labourer in all states of the world. It is worth noting that this constraint will also ensure that the bank is able to meet its debt obligations should the labourer choose to transfer the debt to an artisan within the same region. This is true regardless of whether the bank imposes an intrabank transfer fee or not.

Finally, there is an upper limit on the intrabank transfer fee. As the gain from trade between labourers and artisans is $(1 + \gamma)$, the fee must not be large enough to completely counteract this benefit, as labourers would then choose not to trade with artisans at all. The upper bound on $\chi_{j,j}$ is given by the following inequality.

$$(1 + \gamma) (1 - \chi_{j,j}) \geq 1. \tag{22}$$

The following proposition characterises the equilibrium in autarky.

Proposition 1. *Suppose that bank j does not allow interbank transfers and chooses $n_j, B_j, \chi_{j,j}$ and $\epsilon_{j,j}$ to maximise equation (20) subject to the constraints set out in equations (9), (21), and (22). Then there exists an autarky equilibrium such that it hires labour $n_j^{Autarky}$ where*

$$n_j^{Autarky} = [v (1 + \gamma\alpha)]^{\frac{1}{1-v}}.$$

issues debt equal to $B_j^{Autarky}$ where

$$B_j^{Autarky} = v^{\frac{1}{1-v}} (1 + \gamma\alpha)^{\frac{v}{1-v}},$$

earns profit $V_j^{Autarky}$ where

$$V_j^{Autarky} = v^{\frac{v}{1-v}}(1+\gamma\alpha)^{\frac{v}{1-v}}(1-v).$$

sets the intrabank fee to zero such that $\chi_{j,j} = 0$ and chooses any unit of account such that$\epsilon_{j,j} \leq \frac{1}{v}$.

Proof. See Appendix. □

It is optimal for the intrabank fee to be set to zero, $\chi_{j,j} = 0$. Intuitively, the bank can choose to raise the transfer fee and increase its fee revenue; however, in doing so, it reduces the amount of debt the labourer can transfer to the artisan by the same amount and increases the bank's cost of debt. Given $\gamma > 0$ and there are gains from trade, raising the transfer fee increases the cost of debt more than it increases fee revenue.

Both the quantity of labour chosen by the bank $n_j^{Autarky}$ and the profit it makes, $V_j^{Autarky}$ are less than optimal in autarky whenever $\beta > 0$. This is because by not facilitating interbank transfers, bank j foregoes the benefits that come from trade with foreign artisans. As a consequence, labourers value the bank's debt less and bank j's borrowing capacity is lower than it otherwise would.

In equilibrium, only the labourer's participation constraint, equation (21), binds. Bank j's choice of unit of account for its debt, $\epsilon_{j,j}$, enters the bank's problem only through the promise-keeping constraint, equation (9). It will always be optimal for bank j to choose a unit of account such that this constraint never binds. Intuitively, the bank will always be able to choose to denominate the debt in the same unit as its production output $\hat{\mathbf{b}}_\mathbf{j} = \hat{\mathbf{y}}_\mathbf{j}$ so that $\epsilon_{j,j} = 1$. Should the bank choose to do this, the promise-keeping constraint collapses to $n_j^v \geq B_j$ which would be satisfied so long as the bank makes a non-negative profit. However, this is not the only unit of account that satisfies equation (9) and bank j is able to satisfy this constraint by choosing any unit of account that satisfies $\epsilon_{j,j} \leq \frac{1}{v}$. As $v < 1$ there is some flexibility in bank j's choice of unit of account and they are able to deviate somewhat from $\epsilon_{j,j} = 1$, which can be thought of as bank $j's$ natural unit of account.

3.2 Transferable Debt

Now consider the contract terms that would be offered by bank j if they chose to allow their debt to be transferred to local artisans in region j and foreign artisans in region k. Bank j then chooses labour n_j, the size of the debt B_j, the unit of account $\epsilon_{j,j}$ and the transfer fees. I allow bank j to set different fees for both interbank and intrabank transfers. Bank j chooses a transfer fee $\chi_{j,j} \geq 0$ on local intra-bank transfers and chooses a transfer fee $\chi_{j,k} \geq 0$ for foreign transfers to region k. As discussed earlier, bank j also determines the unit of account for any interbank debt it issues to bank k to facilitate the transfer of bank debt to artisans in region k and that it is optimal for bank j to interbank debt in bank k's unit of account, $\mathbf{b}_k$. As a consequence, the optimal strategy for bank j depends on the choice that bank k makes regarding its unit of account $\mathbf{b}_k$. Before returning to these strategic interactions, I first consider bank j's choice of debt contract, n_j, B, j, $\hat{\mathbf{b}}_j$, $\chi_{j,j}$ and $\chi_{j,k}$, conditional on bank k's unit of account, $\mathbf{b}_k$.

Bank j's optimisation problem is to maximise profit

$$W_j = \max_{\left\{\chi_{j,j},\chi_{j,k},\hat{\mathbf{b}}_\mathbf{j},B_j,n_j\right\}} n_j^v - \left(1 - \alpha\chi_{j,j} - \beta\chi_{j,k}\right) B_j, \tag{23}$$

subject to the participation constraint of the labourer, the promise-keeping constraint on the debt issued to the labourer in region j, the maximum interbank transfer fees that would be paid by the labourer and an additional promise-keeping constraint should the debt be transferred to region k.

The participation constraint of the labourer is similar to that of the autarky case with the addition of the gains from trade that occur from trade with foreign artisans and yields the following equation

$$\left[(1+\gamma)\left(\alpha\left(1-\chi_{j,j}\right)+\beta\left(1-\chi_{j,k}\right)\right)+(1-\alpha-\beta)\right] B_j \geq n_j. \tag{24}$$

The promise-keeping constraint on the original debt contract is identical to that of the autarky case, equation (9). In addition, there is a second promise-keeping constraint relating to the promises bank j makes if it transfers the debt of the labourer to a foreign artisan through an interbank loan to bank k. As discussed earlier, if bank k chooses a different unit of account to bank j, then allowing the debt to be transferable between regions adds what is essentially an exchange rate risk which is defined in equation (10).

Finally, there are upper bounds on the transfer fees. The upper bound on the fee for the intrabank transfer is identical to that in the Autarky case and is given by equation (22). In addition, there is an analogous upper-bound on interbank transfers given by the equation below

$$(1+\gamma)\left(1-\chi_{j,k}\right) \geq 1. \tag{25}$$

Proposition 2. *Suppose that bank j allows interbank transfers and that bank k's unit of account is $\epsilon_{j,k} \leq \frac{1}{v}$. Then both the interbank constraint (10) and the upper-bound on the interbank transfer fee, equation (25) will not bind in equilibrium. Bank j chooses*

$n_j, B_j, \chi_{j,j}$, $\chi_{j,k}$ *and* $\epsilon_{j,j}$ *to maximise equation (20) subject to the constraints set out in equations (9) and (24). Bank* j *issues debt contracts such that*

$$n_j^{(1)} = \left[v\left(1+\gamma(\alpha+\beta)\right)\right]^{\frac{1}{1-v}},$$

$$B_j^{(1)} = v^{\frac{1}{1-v}}\left(1+\gamma(\alpha+\beta)\right)^{\frac{v}{1-v}},$$

and sets both the intrabank and interbank fees to zero so that $\chi_{j,j} = \chi_{j,k} = 0$. *Bank* j *chooses any unit of account such that* $\epsilon_{j,j} \leq \frac{1}{v}$ *and earns profit*

$$W_j^{(1)} = v^{\frac{v}{1-v}}\left(1+\gamma(\alpha+\beta)\right)^{\frac{v}{1-v}}(1-v),$$

Proof. See Appendix. □

This contract is efficient and implements the first best. In this case where bank k chooses $\epsilon_{j,k}$ that is sufficiently small and thus the unit of account of bank k's debt, $\hat{\mathbf{b}}_{\mathbf{k}}$ is sufficiently close to bank j's unit of production $\hat{\mathbf{y}}_{\mathbf{j}}$. In this case, the interbank promise-keeping constraint is relatively easy to satisfy and equation (10) does not bind in equilibrium. Bank j is then able to hire the optimal amount of labour $n_j = n_j^*$ and will choose to allow its debt to be costlessly transferable to region k by setting $\chi_{j,k} = 0$.

Proposition 3. *Suppose that bank* j *allows interbank transfers and bank* k*'s unit of account* $\epsilon_{j,k} > \frac{1}{v}$. *Then bank* j *chooses an interbank transfer fee* $\chi_{j,k} \in [0, \frac{\gamma}{1+\gamma}]$ *according to some function* $\chi^*(\epsilon_{j,k})$ *where* $\chi^{*\prime}(\cdot) \geq 0$.

Bank j *then chooses* $n_j, B_j, \chi_{j,j}$ *and* $\epsilon_{j,j}$ *to maximise equation (20) subject to the constraints set out in equations (9), (10) and (24) and (25) . Bank* j *issues debt contracts such that*

$$n_j^{(2)} = \left[\frac{1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi^*(\epsilon_{j,k})\right)}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right]^{\frac{1}{1-v}},$$

$$B_j^{(2)} = \left(\frac{1}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right)^{\frac{1}{1-v}}\left[1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi^*(\epsilon_{j,k})\right)\right]^{\frac{v}{1-v}},$$

and sets the intrabank fee to zero such that $\chi_{j,j} = 0$.

Bank j *chooses any unit of account such that* $\epsilon_{j,j} \leq \left(1-\chi^*\left(\epsilon_{j,k}\right)\right)\epsilon_{j,k}$ *and earns profit*

$$W_j^{(2)} = \left[\frac{1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi^*(\epsilon_{j,k})\right)}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right]^{\frac{1}{1-v}}\left(1-\frac{1-\beta\chi^*(\epsilon_{j,k})}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right),$$

Proof. See Appendix. □

This contract has a lower investment size than optimal because $\epsilon_{j,k}$ is relatively large, implying that bank k's unit of account on its debt differs too much from bank j's unit of production. This results in the interbank promise-keeping constraint, equation (10) being more difficult to satisfy, and it is no longer feasible to implement the first best debt contract.

In order to ensure that equation (10) is satisfied, bank j first reduces the amount of labour it hires, $n_j < n_j^*$, then as $\epsilon_{j,k}$ increases further, it raises the transfer fee on interbank transfers, setting $\chi_{j,k} > 0$.

As in the autarkic contract, it is always optimal for bank j to set $\chi_{j,j} = 0$. This follows from the fact that the only constraint that $\chi_{j,j}$ enters is the labourer's participation constraint. As in the Autarky case, an increase in $\chi_{j,j}$ tightens this participation cost to a greater extent than the bank benefits from the transfer revenue. As a consequence, it is always more efficient for the bank to lower the cost of debt by setting the fee to zero and increasing the production size rather than gaining transfer fee revenue.

Bank j can always make a choice regarding its own unit of account to ensure that the initial promise-keeping constraint, equation (9), is met by choosing a unit of account close enough to the unit of account of its output. To see this, note that setting $\hat{\mathbf{b}}_{\mathbf{j}} = \hat{\mathbf{y}}_{\mathbf{j}}$ will always ensure that the constraint is slack. The precise range this unit of account can take will depend on the precise form the equilibrium debt contract takes, which in turn depends on the unit of account chosen by the foreign bank, bank k.

Proposition 4. *Suppose that bank j allows interbank transfers and bank k's unit of account $\epsilon_{j,k} > \frac{1}{v}$. Then there exist two cutoffs*

$$\bar{\epsilon}_{j,k}^{(2)} = \frac{1}{v}\left(1 + \beta\left(\frac{\gamma(1-v)(1-\alpha-\beta)}{1+\gamma\alpha-\beta}\right)\right)$$

and

$$\bar{\epsilon}_{j,k}^{(3)} = \frac{1}{v}\left(1 + \gamma + \beta\left(\frac{\gamma(1-v)(1-\alpha)}{1+\gamma\alpha-\beta}\right)\right)$$

where $\frac{1}{v} < \bar{\epsilon}_{j,k}^{(2)} < \bar{\epsilon}_{j,k}^{(3)}$ such that

$$\chi^*(\epsilon_{j,k}) = 0 \iff \epsilon_{j,k} < \bar{\epsilon}_{j,k}^{(2)}$$

and

$$\chi^*(\epsilon_{j,k}) = \frac{\gamma}{1+\gamma} \iff \epsilon_{j,k} \geq \bar{\epsilon}_{j,k}^{(3)}$$

Proof. See Appendix. □

It is not always optimal for bank j to set the interbank transfer fee $\chi_{j,k}$ to zero. This is because $\chi_{j,k}$ enters two constraints, the labourer's participation constraint and the interbank promise-keeping constraint, equation (10). As bank k's unit of account cannot be set directly by bank j, the interbank promise-keeping constraint is not guaranteed to be always slack. However, bank j can relax this constraint by increasing the interbank transfer fee $\chi_{j,k}$. It follows that in cases where bank k chooses a unit of account sufficiently far away from bank j's unit of production $\mathbf{\hat{y}_j}$, it is optimal for bank j to respond by setting higher interbank transfer fees $\chi_{j,k}$.

3.3 Debt transferability

I now characterise the bank's optimal decision over whether to allow its liabilities to circulate beyond its own region. Bank j's decision can be formalised a

$$\pi_j\left(\epsilon_{j,k}\right) = \max\left\{W_j\left(\epsilon_{j,k}\right), V_j^{Autarky}\right\}, \tag{26}$$

where $W_j\left(\epsilon_{i,j}\right)$ denotes bank j's profit if it allows its liabilities to be transferred to region k with

$$W_j\left(\epsilon_{j,k}\right) = \begin{cases} v^{\frac{v}{1-v}}\left(1+\gamma(\alpha+\beta)\right)^{\frac{v}{1-v}}(1-v) & \text{if } \epsilon_{j,k} \leq \frac{1}{v} \\ \left[\frac{1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi^*(\epsilon_{j,k})\right)}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right]^{\frac{1}{1-v}}\left(1-\frac{1-\beta\chi^*(\epsilon_{j,k})}{\epsilon_{j,k}\left(1-\chi^*(\epsilon_{j,k})\right)}\right) & \text{if } \epsilon_{j,k} > \frac{1}{v} \end{cases} \tag{27}$$

and where $V_j^{Autarky}$ is bank j's profit if it chooses autarky as defined in Proposition 1.

The choice bank k makes regarding its unit of account will determine to what extent the interbank promise-keeping constraint binds. The best response of bank j to bank k's decision will be to vary both the terms of the debt contract and to vary whether or not to allow the debt to be transferable to region k or not.

In cases where $\epsilon_{j,k} \leq \frac{1}{v}$, bank j will always choose to allow interbank transfers as their profit is strictly higher than in the autarkic case. However, as $\epsilon_{j,k}$ increases, bank j's profit when allowing interbank transfers. At the limit bank profit goes to zero, $\lim_{\epsilon_{j,k}\to\infty} W_j\left(\epsilon_{j,k}\right) = 0$, and thus bank j earns a higher profit in autarky.

As discussed above, bank j chooses its own unit of account such that the initial borrowing constraint, equation (9), does not bind. Figure 1 shows an illustrative example of the best responses of bank j conditional on bank k's choice of unit of account.

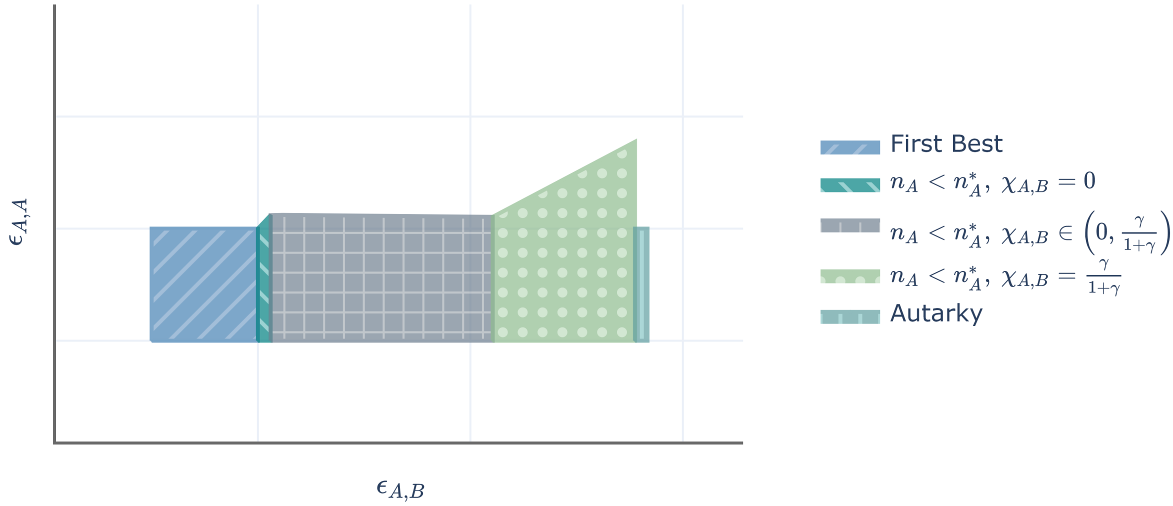

Figure 1: Bank j's Best Responses given Bank k's choice of Unit of Account

Figure 1 provides a useful lens for understanding why a new money issuer may choose to deviate from an established unit of account and in their degree of interoperability with existing forms of money.

Consider the case where bank k is an incumbent bank and that $\epsilon_{j,k}$ represents the existing single unit of account. Assume for now that the incumbent bank does not respond to the new entrant. The new entrant, bank j, chooses its unit of account and transfer policy to maximise its profit. If the incumbent's unit of account differs from the entrant's asset base, the entrant faces a trade-off. It can either align with the incumbent to facilitate interbank transfers and preserve monetary singleness, or it can choose a distinct unit of account that better matches its own unit of production. The model formalises this decision, showing that if $\epsilon_{j,k}$ is sufficiently high, the exchange rate risk increases, and the entrant will rationally restrict interbank transfers, first through higher transfers fees and in extreme cases banning them altogether.

3.4 Strategic interaction of banks

The best response function sets out the unit of account and interoperability choices made by bank j conditional on bank k's unit of account. I consider two types of equilibria. First, a staggered entry equilibrium in which bank k has already chosen its unit of account before the entry of bank j and remains fixed after the entry of bank j. In this equilibrium, while bank k, cannot adjust their unit of account, they are able to choose the degree of transferability that their debt has with bank j. In particular, bank k chooses whether to allow their debt to be transferable to region j and if so whether to charge an interbank transfer fee.

Second, I consider a Nash equilibrium where bank j and k choose units of account simultaneously, taking into account the best response function of the other bank. As bank k faces the symmetric contracting problem to bank j, their choice of unit of account will also be set to ensure their initial promise-keeping constraint does not bind. The Nash equilibria, which can be found as the solution to the best responses of bank j and bank k. In general, since each bank has a region of best responses, there will not be a unique Nash equilibrium.

In the staggered entry equilibrium, I assume that the unit of account of bank k is consistent with bank k's initial autarkic state and thus $\epsilon_{k,k} \leq \frac{1}{v}$ where $\epsilon_{k,k} = \frac{\mathbf{p_k}'\hat{\mathbf{b}}_\mathbf{k}}{\mathbf{p_k}'\hat{\mathbf{y}}_\mathbf{k}}$ is the effective exchange rate from the perspective of bank k. Using the definition of $\epsilon_{k,k}$ and transforming this inequality into $\epsilon_{j,k}$ this implies that bank k sets its unit of account such that

$$\epsilon_{j,k} \geq \frac{1}{v}\left(\frac{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{k} - 2(1-v)}{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{j}}\right). \tag{28}$$

From the perspective of bank k, there is lower-bound $\epsilon_{k,k} \geq 1$ as adopting its unit of production as its unit of account minimises $\epsilon_{k,k}$. This implies an upper-bound on $\epsilon_{j,k}$ such that

$$\epsilon_{j,k} \leq \left(\frac{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{k}}{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{j}}\right). \tag{29}$$

Whether the first best can exist as a staggered entry equilibrium then depends on the value that bank k's unit of account takes. In particular, if $\left(\frac{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{k}}{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{j}}\right) \leq \frac{1}{v}$ then an efficient equilibrium with frictionless interoperability is guaranteed.

I now characterise under which conditions the first best can exist as a Nash equilibrium. In a first best equilibrium, it must be the case that bank k's unit of account is sufficiently close to bank j's unit of output with $\hat{\mathbf{b}}_\mathbf{k} \leq \frac{1}{v}\hat{\mathbf{y}}_\mathbf{j}$. In addition, bank j's unit of account is sufficiently close to bank k's unit of production. This occurs if the following condition is satisfied

$$\left(\frac{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{k} - 2(1-v)}{\mathbf{p_j}'\hat{\mathbf{y}}_\mathbf{j}}\right) \leq 1, \tag{30}$$

where equation (30) is a sufficient condition for the existence of a first best Nash equilibrium and is a less stringent requirement than equation (29) which is a sufficient condition for the existence of a first best staggered equilibrium. This follows from the fact that in a staggered equilibrium, bank k could feasibly fix its unit of account such that $\epsilon_{k,k} = 1$ and this could be inefficient even if a first best Nash equilibrium exists where $\epsilon_{k,k} = \epsilon_{j,j} = 1/v$.

The first best equilibrium exists as a competitive Nash equilibrium if the units of production of the two banks $\hat{\mathbf{y}}_\mathbf{j}$ and $\hat{\mathbf{y}}_\mathbf{j}$ are sufficiently similar, or if the volatility of the relative price of the two tradable goods is not too large. In addition, whenever first-best is feasible, there exists a common unit of account that implements it. By common unit of account, I mean that the bank debt of both banks has the same unit of account and $\hat{\mathbf{b}}_{\mathbf{j}} = \hat{\mathbf{b}}_{\mathbf{k}}$. However, this does not imply that a common unit of account is required for the equilibrium to coincide with the first best. So long as the units of account do not differ too much from each other, the first-best can be implemented.

However, if the variance in the relative price of the two tradable goods is sufficiently high and the units of production for the two banks are too distinct, the set of equilibria will be inefficient. In particularly extreme cases, an autarkic equilibrium could exist where both banks choose very distinct units of account and restrict their debt to be locally transferable only. This autarkic equilibrium is the most inefficient type of equilibrium as the economy is denied the gains from trade between labourers and artisans of distinct regions. I will discuss possible policy interventions that can help promote an efficient equilibrium in section 5.

4 Numerical Examples

In this section, I illustrate the range of competitive Nash equilibria that may exist and how this can depend on the variance of the relative prices for the tradable goods. To fix ideas, I assume that bank j produces only good A and bank k produces only good B. I adopt a helpful abuse of notation in which I refer to banks by the good they produce. Thus, I replace the subscript j with the subscript A and the subscript k with the subscript B. Thus, the units of production for the two banks is

$$\hat{\mathbf{y}}_{\mathbf{A}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \hat{\mathbf{y}}_{\mathbf{B}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{31}$$

I also simplify the price vector so that there are only two states that may occur with equal probability. In state A $p_j = 1 - \rho_A$ and $p_B = 1 + \rho_B$ and in state B $p_A = 1 + \rho_A$ and $p_B = 1 - \rho_B$ where ρ_A and ρ_B are normalised to be positive and capture the variance in the two prices.

Figure 3 illustrates the best responses of both banks when the price variance is relatively low and where $\rho_A = \rho_B = 0.4$. The set of competitive Nash equilibria is simply the region where the best response functions overlap. The figure highlights a subset of this region outlined by a grey dotted line where the Nash equilibria implement the first best allocation. This figure also illustrates that there exists a set of competitive Nash equilibria

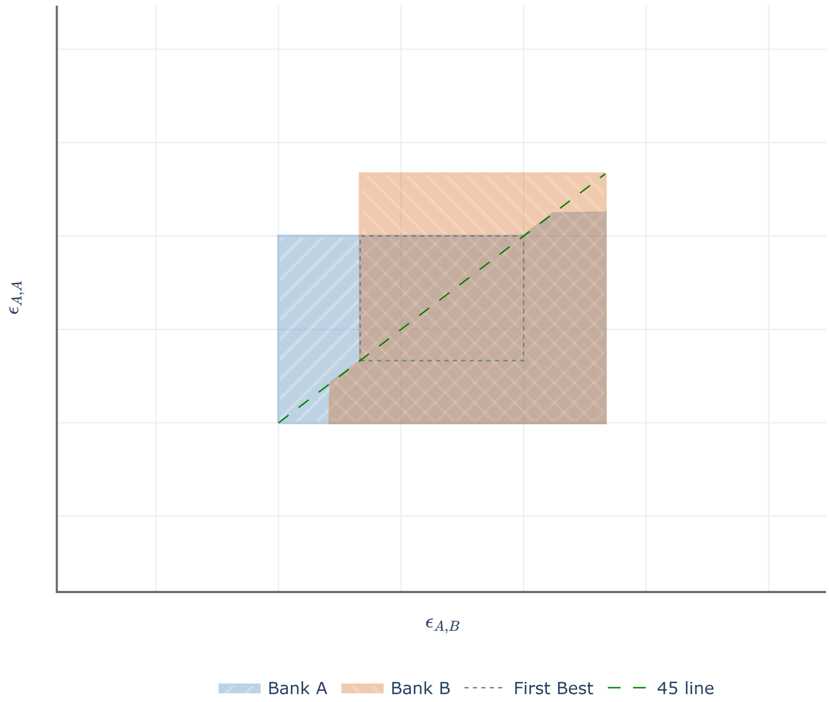

Figure 2: Bank Best Responses ($\rho_A = \rho_B = 0.4$)

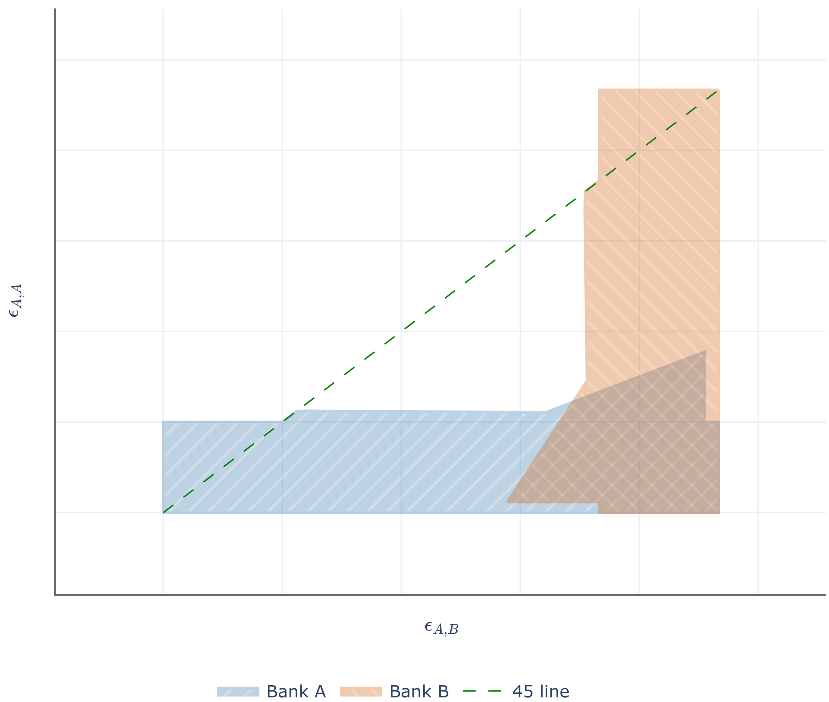

Figure 3: Bank Best Responses ($\rho_A = \rho_B = 0.7$)

that do not implement the first-best equilibria. Finally, the green-dashed 45 degree line highlights the possible common units of account. It should be noted that, while many Nash equilibria with a common unit of account are first-best efficient, a common unit of account is not in of itself a guarantee of efficiency. In this example, there are some inefficient Nash equilibria that also have a common unit of account. However, one benefit of a common unit of account that is worth highlighting is that having a common unit of account rules out equilibria where banks charge a fee on interbank transfers. Intuitively, if banks use a common unit of account, there is no uncertainty regarding the exchange rate of bank debt at the point of transfer.

Figure 3 illustrates the best responses of both banks when the price variance is relatively high and where $\rho_A = \rho_B = 0.7$. In this case, the only feasible competitive Nash equilibria are inefficient. In this case, two types of equilibria are feasible. In one possible case, bank debt can be transferred between regions, but banks charge the maximum possible fee, $\chi_{A,B} = \chi_{B,A} = \frac{\gamma}{1+\gamma}$. The other possible case is the autarkic case where both banks restrict the transferability of their debt to local intrabank transfers. It is also worth noting that in this example, where price variance is high, no possible equilibria feature a common unit of account.

5 Discussion

In this section, I discuss the effectiveness of various tools available to policy makers who wish to promote an efficient equilibrium as well as other policy implications of the model.

5.1 Mandating A Common Unit of Account

I now consider the concept of a common unit of account. The model suggests that, while there are benefits to pursing a common unit of account, there are risks in mandating a common unit of account. Rather, it would be best for a policy maker to create an environment in which a common unit of account arises naturally as the result of a competitive equilibrium.

First, the existence of a common unit of account rules out equilibria where the banks charge transfer fees on interbank transfers. This is because a common unit of account removes the interest risk on interbank transfers that can result in banks choosing to charge transfer fees. However, as illustrated by the example set out in figure 3, a common unit of account does not guarantee an efficient equilibrium.

In cases where there are no Nash equilibria with a common unit of account, mandating a common unit of account would result in banks acting suboptimally. Promoting a common unit account is rather one tool a policy maker can use, but it is best supplemented by other tools such as central bank reserves and the availability of cash.

To fix ideas, consider a case where the unit of account is mandated to be the unit of production of an incumbent bank k. From bank k's perspective, its initial promise-keeping constraint will always hold when $\hat{\mathbf{b}}_{\mathbf{k}} = \hat{\mathbf{y}}_{\mathbf{k}}$ thus this mandated unit of account appears beneficial to them. Consider now the problem of an entrant bank j. Their problem remains as it did before, except that they are subject to the following additional constraint:

$$\hat{\mathbf{b}}_{\mathbf{j}} = \hat{\mathbf{y}}_{\mathbf{k}}. \tag{32}$$

Substituting this constraint into their promise-keeping constraints, equations (9) and (10) show that there is no benefit now in setting $\chi_{j,k} > 0$. It follows that bank j sets its interbank transfer fees to zero. It is also worth noting that if a common unit of account is mandated even if bank j were to choose autarky, then they would be strictly better off allowing interbank transfers. Mandating a common unit of account thus has the benefit that it encourages full interoperability between banks: neither bank has an incentive to restrict interbank transfers or to charge interbank transfer fees. However, bank j must still satisfy its promise-keeping constraints, equations (9) and (10). If it cannot satisfy these constraints by adjusting its own unit of account or by reducing interoperability, the only way that bank j can ensure that these constraints are satisfied is by reducing its borrowing amount n_j to a sub-optimal level.

Furthermore, it follows from Figure 2 that in cases where a Nash equilibrium with a common unit of account exists, it is not necessarily the only optimal equilibrium. In these cases while a common unit of account often coincides with first-best equilibria, it is not a necessary condition for efficiency. Thus, this paper suggests that small deviations from singleness could be tolerated by policymakers, especially in cases where households may benefit from features of the different forms of money.

5.2 Central Bank Reserves

I now consider a simple policy that captures the role of central bank reserves in the banking system. Consider a government that chooses to tax a fraction $\tau \in [0, 1]$ of the output of the two banks. I assume that the tax rate is announced at date 0 and collected at date 1. I assume that the government does not consume any of the tax revenues itself, but instead will transfer a fraction of the tax revenue to each bank proportional to the total amount of tax paid by each bank. I assume that the government issues each bank at date 0 a tax bill which requires each bank to commit to pay the required tax and a receipt of repayment that commits the government to providing the transfer at date 2.

This intervention is ex ante neutral, as each bank receives a transfer equal in expected value to the tax payment they are required to make. However, taxes and transfers will differ in their units of account. Tax payments are assumed to be made in real terms, so that the unit of account of the tax payment bank j has a unit of account $\hat{\mathbf{y}}_{\mathbf{j}}$. The transfer, on the other hand, in this simple example is denoted by $\hat{\mathbf{y}}_{\mathbf{m}}$ where

$$\hat{\mathbf{y}}_{\mathbf{m}} = \frac{1}{2}\left(\hat{\mathbf{y}}_{\mathbf{j}} + \hat{\mathbf{y}}_{\mathbf{k}}\right). \tag{33}$$

In this way, the government can be thought of as introducing, through a very simple system of taxes and transfers, fiat money and a monetary unit of account $\hat{\mathbf{y}}_{\mathbf{m}}$, and the operation can be thought of as the introduction of central bank reserves into the system.

Now consider what this simple policy implies for the set of competitive equilibria. Bank j's initial endowment at the beginning of date 2 now consists of their production output net of taxes and transfers. The unit of production for bank j after taxes and transfers is thus

$$\tilde{\mathbf{y}}_{\mathbf{j}} = (1-\tau)\,\hat{\mathbf{y}}_{\mathbf{j}} + \tau\hat{\mathbf{y}}_{\mathbf{m}}. \tag{34}$$

The introduction of central bank reserves can be thought of as reducing the variance in prices of the banks' output. As highlighted earlier, this increases the likelihood that an efficient equilibrium exists and reduces the likelihood that an inefficient autarkic equilibrium exists.

In this simple setting, the policy maker can also guarantee the existence of a unique efficient equilibrium by setting the tax rate $\tau = 1$. The simple model of this paper abstracts from any of the negatives that a 100% tax rate may introduce, but this result can perhaps be thought of as an argument in favour of narrow banking.

5.3 Cash

I now turn to the role cash may have in this framework. Cash is a form of fiat money that can be used for transaction purposes. I thus introduce cash in the following way. At date 1, the labourer can request that the bank provide it with cash. I assume that all artisans in all regions accept cash as a means of payment. I assume that on demand, the bank obtains cash from the central bank in exchange for a claim on its future output.

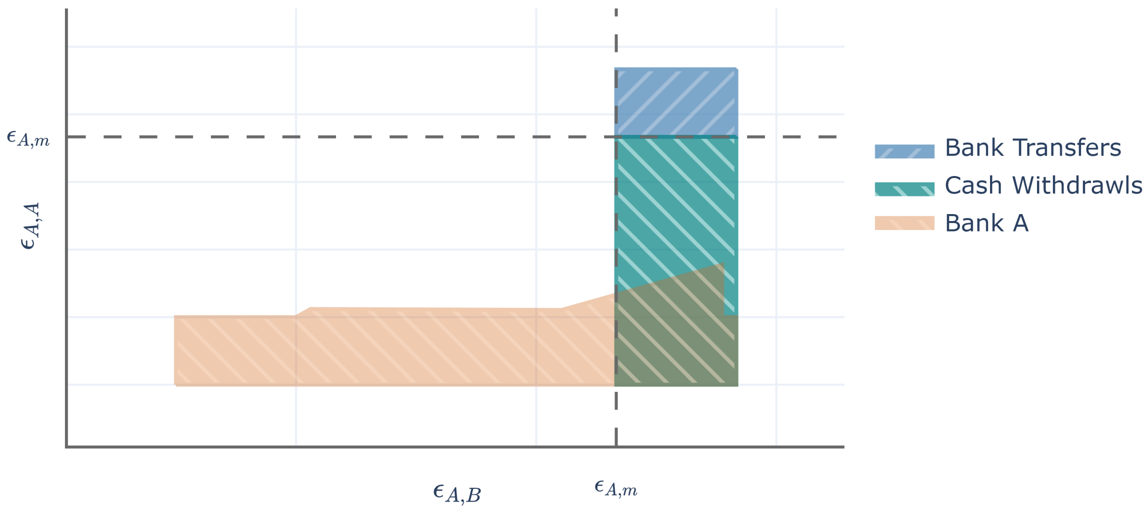

Figure 4: Bank Best Response functions with cash ($\epsilon_{j,m} = \frac{1}{v}$)

The bank issues debt to the central bank in the same way as it would issue debt to a foreign bank on the interbank market. The only difference is that, rather than having to exchange debt in the foreign unit of account, the bank exchanges to the monetary authority unit of account $\hat{\mathbf{y}}_{\mathbf{m}}$.

The bank is thus able to benefit from the gains of trade between labourers and foreign artisans if it allows bank transfers or cash withdrawals. Should it do so, bank j's optimisation problem is altered so that equation (10) can be rewritten as

$$n_j^v \geq B_j \min \left\{ \left(1 - \chi_{j,k}\right) \epsilon_{j,k}, \epsilon_{j,m} \right\}, \tag{35}$$

where $\epsilon_{j,m} = \frac{\mathbf{p_j}'\hat{\mathbf{b}}_m}{\mathbf{p_j}'\hat{\mathbf{y}}_j}$.

Cash places an effective upper bound on the effect the foreign bank's unit of account has on the bank's best response function. If the foreign bank chooses a unit of exchange that is far from the bank's unit of production to the extent that the interbank promise-keeping constraint binds, the bank can choose not to allow interbank transfers but to instead allow cash withdrawals.

Although cash could play a bridging role if $\hat{\mathbf{b}}_m$ lies between the two units of account chosen by banks j and k, this would suggest a change in the national unit of account following the entry of bank j, which seems unlikely. However, cash can play an additional role in limiting the impact of new forms of money on an incumbent bank. Consider the case where bank k adopts the monetary unit of account such that $\hat{\mathbf{b}}_k = \hat{\mathbf{b}}_m$ while bank j enters with a distinct unit of account such that $\epsilon_{k,j} \geq \frac{1}{v}$. Absent cash, bank k would limit interoperability with bank j, either by charging interbank fees or banning transfers altogether. This is inefficient, as bank k's depositors cannot fully benefit from trade with bank j's depositors. If cash exists as a universally accepted medium of exchange, bank k can facilitate trade between its depositors and bank k's depositors through cash withdrawals. In this scenario, bank k operates at its efficient scale.

In the case where $\hat{\mathbf{b}}_k = \hat{\mathbf{b}}_m$ only the incumbent bank benefits from the presence of cash. Bank j will be indifferent between allowing partial interoperability with cash or with bank k directly.

Figure 4 illustrates the best response of the two banks, as the policy maker introduces cash with a unit of account $\mathbf{p_j}'\hat{\mathbf{b}}_m$. In this case, bank k will always be able to achieve the optimal investment level regardless of bank j's choice of unit of account. On the other hand, bank j will limit interoperability with both cash and bank k and choose a unit of account sufficiently far from bank k's.

5.4 Innovation and the unit of account

As in Doepke and Schneider (2017), an important foundation for efficiency is low relative price volatility. Thus, low inflation is not an important prerequisite for the existence of an efficient equilibrium. What matters instead is the similarity of the business model of the banks and their underlying risk. This is an important point to consider in light of recent developments in the payment landscape. New forms of digital money, for example, stablecoins and tokenised deposits, have already been developed or are likely to be developed in the near future.

The model suggests that policymakers should carefully monitor the development and adoption of these new forms of digital money. In the context of the model, banks located in the same country are likely to have similar asset portfolios, and thus are likely to adopt a common unit of account. New forms of digital money bring the possibility that money could be backed with assets that may be more diverse than a traditional deposit taking bank. For example, a stablecoin backed by foreign assets or launched by Big Tech could have an underlying asset base that differs from those of traditional banks, making the likelihood of an autarkic equilibrium more likely.

6 Conclusion

Rapid innovation in digital payments and the advent of new forms of privately issued digital money have increased the interest of central banks in the singleness of money and the maintenance of a common unit of account. This paper has developed a tractable model to explore the conditions under which singleness can be preserved and the consequences when it is not.

A key insight of this paper is that singleness of money is an equilibrium outcome shaped by institutional design, market structure, and policy choices. When banks issue liabilities in divergent units of account and when interbank transfers are costly or non-existent, the economy risks slipping into inefficient equilibria. These frictions can fragment the monetary system, reduce the efficiency of trade, and ultimately erode welfare.

The model shows that, while a common unit of account often supports first-best outcomes, it is not strictly necessary. Small deviations from a shared unit of account can be tolerated, particularly when supported by robust institutional mechanisms such as central bank reserves and access to cash. These tools act as stabilisers, reducing the variance in banks' asset bases and providing fallback mechanisms for payment interoperability. In this sense, cash may act as a strategic backstop that anchors the monetary system when interoperability of private money breaks down.

The policy implications are clear. First, central banks should remain vigilant to the entry of new forms of digital money, especially those with business models and backing assets that differ significantly from incumbent banks. Stablecoins backed by foreign assets or issued by Big Tech platforms may pose particular risks to monetary cohesion.

Second, policymakers should design regulatory frameworks that encourage convergence in units of account, without mandating singleness. A competitive equilibrium that naturally yields a common unit of account is more resilient than one imposed by fiat.

Third, the model provides a theoretical foundation for a pragmatic approach to monetary innovation. Rather than insisting on perfect singleness, policy makers can aim for "functional singleness", a regime in which small deviations are tolerated, provided that they do not affect the efficiency of payments or the interoperability of different types of money.

Finally, the paper highlights the importance of understanding the microeconomic foundations of money. The unit of account is not merely a numeraire; it is a strategic choice that shapes the feasibility of contracts, the efficiency of trade, and the architecture of the monetary system.

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Appendix

Bank j's problem without interbank transfers

The Lagrangian is

$$\begin{aligned}\mathcal{L}_j^{Autarky} =& n_j^v - (1 - \alpha\chi_{j,j} \\ &+ \lambda_N \left[((1+\gamma)(\alpha(1-\chi_{j,j}) + 1 - \alpha - \beta)B_j - n_j\right] \\ &+ \mu_j^+ \left[(1+\gamma)(1-\chi_{j,j}) - 1\right] \\ &+ \mu_j^- \chi_{j,j}\end{aligned}$$

We take the first-order conditions with respect to $n_j, B_j, \chi_{j,j}$ which are as follows:

$$\frac{\partial \mathcal{L}_j^{Autarky}}{\partial n_j} : \quad v n_j^{v-1} - \lambda_N = 0$$

$$\frac{\partial \mathcal{L}_j^{Autarky}}{\partial B_j} : \quad \lambda_N \left[(1+\gamma)\alpha(1-\chi_{j,j}) + 1 - \alpha\right] - (1 - \alpha\chi_{j,j}) = 0$$

$$\frac{\partial \mathcal{L}_j^{Autarky}}{\partial \chi_{j,j}} : \quad \alpha B_j - \lambda_N(1+\gamma)\alpha B_j - \mu_j^+(1+\gamma) + \mu_j^- = 0$$

It follows from the latter equation that

$$\chi_{j,j} \iff \lambda^N > \frac{1}{1+\gamma}$$

Rearranging the second equation yields

$$\lambda_N = \frac{1 - \alpha\chi_{j,j}}{(1+\gamma)\alpha(1-\chi_{j,j}) + 1 - \alpha}$$

where we note that this implies $\lambda^N > \frac{1}{1+\gamma}$ for any value of $\chi_{j,j} \geq 0$.

Thus we note that it is optimal in autarky for firms not charge for intrabank transfers.

Substituting out λ_N in the first equation yields an equation for the quantity of labour hired in autarky

$$n_j^{Autarky} = \left[v\left(1 + \gamma\alpha\right)\right]^{\frac{1}{1-v}}.$$

The loan size follows from the budget constraint

$$B_j^{Autarky} = v^{\frac{1}{1-v}} (1+\gamma\alpha)^{\frac{v}{1-v}} ,$$

and the profit to bank j is

$$\pi_j^{Autarky} = v^{\frac{v}{1-v}}(1+\gamma\alpha)^{\frac{v}{1-v}}(1-v).$$

Finally, note that the bank's unit of account can be set to any value that ensures the promise keeping condition described by equation (3) holds. This is satisfied so long as

$$\epsilon_{j,j} \leq \frac{1}{v}.$$

Bank j's problem with interbank transfers

Note we drop the promise-keeping constraint as Bank A can simply choose its own unit of account to ensure it does not bind.

The Lagrangian then becomes

$$\begin{aligned}
\mathcal{L}_j =& n_j^v - (1-\alpha\chi_{j,j} - \beta\chi_{j,k})B_j \\
&+ \lambda_N \left[((1+\gamma)(\alpha(1-\chi_{j,j}) + \beta(1-\chi_{j,k})) + 1 - \alpha - \beta)B_j - n_j\right] \\
&+ \lambda_k \left[\frac{n_j^v}{\epsilon_{j,k}} - (1-\chi_{j,k})B_j\right] \\
&+ \mu_j^+ \left[(1+\gamma)(1-\chi_{j,j}) - 1\right] \\
&+ \mu_k^+ \left[(1+\gamma)(1-\chi_{j,k}) - 1\right] \\
&+ \mu_j^- \chi_{j,j} \\
&+ \mu_k^- \chi_{j,k}
\end{aligned} \tag{A.36}$$

We take the first order conditions with respect to $n_j, B_j, \chi_{j,j}, \chi_{j,k}$ which are as follows

$$\frac{\partial \mathcal{L}_j}{\partial n_j} : \quad v n_j^{v-1}(1 + \frac{1}{\epsilon_{j,k}}\lambda_k) - \lambda_N = 0 \tag{A.37}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}_j}{\partial B_j} : \quad & \lambda_N \left[(1+\gamma)(\alpha(1-\chi_{j,j}) + \beta(1-\chi_{j,k})) + 1 - \alpha - \beta\right] \\
& - (1-\alpha\chi_{j,j} - \beta\chi_{j,k}) - \lambda_k(1-\chi_{j,k}) = 0
\end{aligned} \tag{A.38}$$

$$\frac{\partial \mathcal{L}_j}{\partial \chi_{j,j}}: \quad \alpha B_j - \lambda_N(1+\gamma)\alpha B_j - \mu_j^+(1+\gamma) + \mu_j^- = 0 \tag{A.39}$$

$$\frac{\partial \mathcal{L}_j}{\partial \chi_{j,k}}: \quad \beta B_j - \lambda_N(1+\gamma)\beta B_j + \lambda_k B_j - \mu_k^+(1+\gamma) + \mu_k^- = 0 \tag{A.40}$$

B Proof of Proposition 2

First assume that $\lambda_k = 0$ and later verify under which conditions this holds.

From rearranging equation (A.39) it follows that

$$\chi_{j,j} = 0 \iff \lambda_N > \frac{1}{1+\gamma}. \tag{B.1}$$

Similarly, rearranging equation (A.40) yields

$$\chi_{j,k} = 0 \iff \lambda_N > \frac{\beta + \lambda_k}{(1+\gamma)\beta}. \tag{B.2}$$

Note that in the case where $\lambda_k = 0$ equations (B.1) and (B.2) are equivalent.

Rearranging equation (A.38) yields the following equation for λ_N

$$\lambda_N = \frac{1 - \alpha\chi_{j,j} - \beta\chi_{j,k} + \lambda_k(1-\chi_{j,k})}{1 + \gamma(\alpha+\beta) - (1+\gamma)(\alpha\chi_{j,j} + \beta\chi_{j,k})} \tag{B.3}$$

and note that in the case where $\lambda_k = 0$, equations (B.1) and (B.2) imply that $\chi_{j,j} = \chi_{j,k} = 0$.

Now substituting out λ_N from equation (A.37) and rearranging yields

$$n_j = \left[v\left(1 + \gamma(\alpha+\beta)\right)\right]^{\frac{1}{1-v}}. \tag{B.4}$$

In equilibrium, the labourer's participation constraint, equation (24) holds with strict equality and the debt size immediately follows

$$B_j = v^{\frac{1}{1-v}}\left(1 + \gamma(\alpha+\beta)\right)^{\frac{v}{1-v}}. \tag{B.5}$$

Substituting the equations for n_j and B_j into bank j's profit function yields the profit function

$$W_j = v^{\frac{v}{1-v}}(1+\gamma(\alpha+\beta))^{\frac{v}{1-v}}(1-v). \tag{B.6}$$

Next, note that in order for the bank's initial promise-keeping constraint, equation (9) to be satisfied, it is required that

$$\epsilon_{j,j} \leq \frac{1}{v}. \tag{B.7}$$

Finally, it remains to find the conditions under which $\lambda_k = 0$. For this to hold it must be the case that the interbank promise-keeping constraint, equation (10) is slack. Substituting the equations for n_j and B_j into equation (10) yields the following inequality

$$\epsilon_{j,k} \leq \frac{1}{v}. \tag{B.8}$$

C Proof of Proposition 3

Consider now the case where $\epsilon_{j,k} > \frac{1}{v}$ and thus $\lambda_k > 0$. Here, equation (B.1) is still satisfied and $\chi_{j,j} = 0$.

Rearranging the first order conditions, and binding constraints yields the following system of equations

$$\lambda_N = \frac{1+(1-\chi_{j,k})\lambda_k}{1+\gamma\alpha+\beta(\gamma-(1+\gamma)\chi_{j,k})} \tag{C.1}$$

$$n_j^{v-1}\frac{1}{\epsilon_{j,k}}(1+\gamma\alpha+\beta(\gamma-(1+\gamma)\chi_{j,k})) = (1-\chi_{j,k}) \tag{C.2}$$

and

$$\lambda_N = vn_j^{v-1}(1+\lambda_k\frac{1}{\epsilon_{j,k}}). \tag{C.3}$$

Solving this system of equations for n_j yields

$$n_j = \left[\frac{1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi_{j,k}\right)}{\epsilon_{j,k}\left(1-\chi_{j,k}\right)}\right]^{\frac{1}{1-v}} \tag{C.4}$$

The debt level follows from rearranging equation (24)

$$B_j = \frac{n_j}{1+\gamma\alpha+\beta(\gamma-(1+\gamma)\chi_{j,k})} \tag{C.5}$$

which yields

$$B_j = \left(\frac{1}{\epsilon_{j,k}\left(1-\chi_{j,k}\right)}\right)^{\frac{1}{1-v}} \left[1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi_{j,k}\right)\right]^{\frac{v}{1-v}}. \tag{C.6}$$

Solving for bank j's profit yields

$$W_j = \left[\frac{1+\gamma\alpha+\beta\left(\gamma-(1+\gamma)\chi_{j,k}\right)}{\epsilon_{j,k}\left(1-\chi_{j,k}\right)}\right]^{\frac{1}{1-v}} \left(1-\frac{1-\beta\chi_{j,k}}{\epsilon_{j,k}\left(1-\chi_{j,k}\right)}\right) \tag{C.7}$$

D Proof of Proposition 4

Consider equation (A.40) in the case where $\lambda_k > 0$.

There are three possible scenarios to consider.

First, consider the case where $\chi_{j,k} = 0$. This occurs if

$$\frac{1+\lambda_k}{1+\gamma(\alpha+\beta)} > \frac{\beta+\lambda_k}{(1+\gamma)\beta} \tag{D.1}$$

which simplifies to

$$\lambda_k < \frac{\gamma\beta(1-\alpha-\beta)}{1-\beta+\gamma\alpha}. \tag{D.2}$$

Solving for λ_k in the case where $\chi_{j,k} = 0$ yields

$$\lambda_k = \frac{v\epsilon_{j,k}-1}{(1-v)} \tag{D.3}$$

and combining these last two equations and rearranging yields the result that

$$\chi_{j,k} = 0 \iff \epsilon_{j,k} \leq \frac{1}{v}\left(1+\beta\left(\frac{\gamma(1-v)(1-\alpha-\beta)}{1+\gamma\alpha-\beta}\right)\right) \equiv \bar{\epsilon}^{(2)}_{j,k}. \tag{D.4}$$

Similarly, consider the case where the maximum interbank transfer fee is charged, that is $\chi_{j,k} = \frac{\gamma}{1+\gamma}$. This occurs if

$$\frac{1+\frac{1}{1+\gamma}\lambda_k}{1+\gamma\alpha} < \frac{\beta+\lambda_k}{(1+\gamma)\beta} \tag{D.5}$$

Solving for λ_k in the case where $\chi_{j,k} = \frac{\gamma}{1+\gamma}$ yields

$$\lambda_k = \frac{v\epsilon_{j,k}-(1+\gamma)}{1-v} \tag{D.6}$$

and combining these last two equations and rearranging yields the result that

$$\chi_{j,k} = \frac{\gamma}{1+\gamma} \iff \epsilon_{j,k} \geq \frac{1}{v}\left(1+\gamma+\beta\left(\frac{\gamma(1-v)(1-\alpha)}{1+\gamma\alpha-\beta}\right)\right) \equiv \bar{\epsilon}^{(3)}_{j,k}. \tag{D.7}$$

If $\bar{\epsilon}^{(2)}_{j,k} < \epsilon_{j,k} < \bar{\epsilon}^{(3)}_{j,k}$ then $\chi_{j,k} \in \left(0, \frac{\gamma}{1+\gamma}\right)$ can be found from the solution to the following quadratic equation

$$\begin{aligned}&(1+\gamma\alpha-\beta) + (1-v)(1-\chi_{j,k})(\beta^2+\gamma\beta(1-\alpha)) \\ &= (1-v)(1-\chi_{j,k})^2\beta^2(1+\gamma) + v(1-\chi_{j,k})(1+\gamma\alpha-\beta)\epsilon_{j,k}.\end{aligned} \tag{D.8}$$